

峨眉二中 21 级高一下半期考试数学科试题（理）

参考答案

1	2	3	4	5	6	7	8	9	10	11	12
B	B	C	C	B	A	D	A	D	B	C	B

$$12. \frac{1}{2}a_n^2 = a_{n+1} - a_n, 2a_{n+1} = a_n^2 + 2a_n, \frac{1}{2a_{n+1}} = \frac{1}{a_n^2 + 2a_n} = \frac{1}{2}\left(\frac{1}{a_n} - \frac{1}{a_n + 2}\right),$$

$$\frac{1}{a_n + 2} = \frac{1}{a_n} - \frac{1}{a_{n+1}}, b_n = \frac{1}{a_n + 2} + 1 = \frac{1}{a_n} - \frac{1}{a_{n+1}} + 1,$$

$$T_{2021} = \frac{1}{a_1} - \frac{1}{a_2} + 1 + \frac{1}{a_2} - \frac{1}{a_3} + 1 + \dots - \frac{1}{a_{2021}} - \frac{1}{a_{2022}} + 1 = 2021 + \frac{1}{a_1} - \frac{1}{a_{2022}} = 2023 - \frac{1}{a_{2022}},$$

$$a_{n+1} - a_n = \frac{1}{2}a_n^2 \geq 0, a_{n+1} \geq a_n, \therefore a_n \geq \frac{1}{2}, a_{n+1} = a_n + \frac{1}{2}a_n^2 \geq a_n + \frac{1}{4}a_n = \frac{5}{4}a_n$$

$$a_{2022} \geq a_1 \cdot \left(\frac{5}{4}\right)^{2021} > 1, 0 < \frac{1}{a_{2022}} < 1, \therefore T_{2021} = 2023 - \frac{1}{a_{2022}} \in (2022, 2023),$$

所以整数部分是 2022，选 B

$$13. 6 + 6\sqrt{3} \quad 14.70 \quad 15. a_n = e^{n-1} \quad 16.1$$

$$17. \text{解 (1)} \because |\vec{a}|=2, |\vec{b}|=1, (\vec{a} - 3\vec{b}) \cdot (\vec{a} + \vec{b}) = 3,$$

$$\therefore 2^2 - 3 \times 1^2 - 2\vec{a} \cdot \vec{b} = 3, \text{解得 } \vec{a} \cdot \vec{b} = -1. \cos \theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} = -\frac{1}{2}, \theta = \frac{2\pi}{3} \quad (5 \text{ 分})$$

$$(2) |\vec{a} + \vec{b}| = \sqrt{(\vec{a} + \vec{b})^2} = \sqrt{2^2 + 2 \times (-1) + 1^2} = \sqrt{3}; \quad (10 \text{ 分})$$

$$18. \text{解: (1) 当 } a = -1 \text{ 时, 不等式化为 } -x^2 - 4x - 3 < 0, \text{ 即 } x^2 + 4x + 3 > 0,$$

$$\text{解得 } x < -3 \text{ 或 } x > -1, \quad (4 \text{ 分})$$

$$\text{所以不等式的解集为 } \{x | x < -3 \text{ 或 } x > -1\}; \quad (6 \text{ 分})$$

$$(2) \text{ 当 } a = 0 \text{ 时, 不等式化为 } -3 < 0 \text{ 在 } \mathbf{R} \text{ 上恒成立, 符合题意; } \quad (8 \text{ 分})$$

$$\text{当 } a \neq 0 \text{ 时, 因为关于 } x \text{ 的一元二次不等式 } ax^2 + 4ax - 3 < 0 \text{ 的解集为 } \mathbf{R},$$

$$\text{所以 } \begin{cases} a < 0 \\ \Delta = 16a^2 + 12a < 0 \end{cases}, \quad (10 \text{ 分})$$

解得 $-\frac{3}{4} < a < 0$.

综上, a 的取值范围是 $(-\frac{3}{4}, 0]$. (12 分)

19. (1) $\because a_4^2 = a_2 \cdot a_8, \therefore (a_1 + 6)^2 = (a_1 + 2) \cdot (a_1 + 14),$

$$\therefore a_1 = 2, a_n = 2 + (n-1) \times 2 = 2n \quad (6 \text{ 分})$$

$$(2) \frac{1}{a_n a_{n+1}} = \frac{1}{2n \cdot 2(n+1)} = \frac{1}{4n(n+1)} = \frac{1}{4} \left(\frac{1}{n} - \frac{1}{n+1} \right)$$

$$\begin{aligned} S_n &= \frac{1}{4} \left(1 - \frac{1}{2} \right) + \frac{1}{4} \left(\frac{1}{2} - \frac{1}{3} \right) + \frac{1}{4} \left(\frac{1}{n} - \frac{1}{n+1} \right) \\ &= \frac{1}{4} \left(1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \cdots + \frac{1}{n} - \frac{1}{n+1} \right) = \frac{1}{4} \left(1 - \frac{1}{n+1} \right) = \frac{n}{4(n+1)} \end{aligned} \quad (12 \text{ 分})$$

20. 解: (1) $\triangle ABD$ 中, $\angle A = 60^\circ$, $\cos \angle ABD = -\frac{1}{7}$.

所以 $\sin \angle ABD = \frac{4\sqrt{3}}{7}$,

所以 $\sin \angle ADB = \sin (60^\circ + \angle ABD) = \frac{\sqrt{3}}{2} \times \left(-\frac{1}{7} \right) + \frac{1}{2} \times \frac{4\sqrt{3}}{7} = \frac{3\sqrt{3}}{14}$; (6 分)

(2) $\triangle ABD$ 中, 由正弦定理得, $\frac{3}{\sin \angle ADB} = \frac{BD}{\sin 60^\circ}$,

所以 $BD = \frac{3 \times \frac{\sqrt{3}}{2}}{\frac{3\sqrt{3}}{14}} = 7$,

又 $\angle ADC = 90^\circ$,

所以 $\cos \angle BDC = \sin \angle ADB = \frac{3\sqrt{3}}{14}$,

因为 $\triangle BDC$ 的面积 $S = \frac{1}{2} BD \cdot CD = \frac{1}{2} \times 7CD \times \frac{13}{14} = \frac{39\sqrt{3}}{4}$,

所以 $CD = 3\sqrt{3}$. (12 分)

21. 解: (1) $\vec{m} \cdot \vec{n} = \sin A \cdot \cos B + \sin B \cdot \cos A = \sin(A+B) \cdots (2 \text{ 分})$

对于 $\triangle ABC$, $A+B = \pi - C$, $0 < C < \pi$, $\therefore \sin(A+B) = \sin C$, $\therefore \vec{m} \cdot \vec{n} = \sin C$. $\cdots (3 \text{ 分})$

又 $\vec{m} \cdot \vec{n} = \sin 2C$, $\therefore \sin 2C = \sin C$, $\cos C = \frac{1}{2}$, $\therefore C = \frac{\pi}{3}$. $\cdots (6 \text{ 分})$

(2) 由 $\sin A$, $\sin C$, $\sin B$ 成等差数列, 得 $2\sin C = \sin A + \sin B$,

由正弦定理得 $2c = a + b$. $\cdots (8 \text{ 分})$

$$\therefore \overrightarrow{CA} \cdot \overrightarrow{CB} = 18, \therefore ab \cos C = 18, \therefore ab = 36. \dots (10 \text{ 分})$$

由余弦定理 $c^2 = a^2 + b^2 - 2ab \cos C = (a+b)^2 - 3ab$, 可得 $c^2 = 4c^2 - 3 \times 36$,

$$\therefore c^2 = 36, \text{ 解得 } c = 6. \dots (12 \text{ 分})$$

22. 解: (1) 设各项均不相等的等差数列 $\{a_n\}$ 的公差为 $d \neq 0$, $\because a_1 = 1$, 且 a_1, a_2, a_5 成等比数列,

$$\therefore a_2^2 = a_1 \cdot a_5, \text{ 即 } (1+d)^2 = 1+4d, \text{ 解得 } d=2,$$

$$\therefore a_n = 1 + 2(n-1) = 2n-1. (3 \text{ 分})$$

(2) 证明: 数列 $\{b_n\}$ 中, $b_1 = \log_2(a_2+1) = \log_2 4 = 2$,

$$\because b_{n+1} = 4b_n + 2^{n+1}, n \in \mathbb{N}^*.$$

$$\therefore b_{n+1} + 2^{n+1} = 4(b_n + 2^n), b_1 + 2 = 4.$$

\therefore 数列 $\{b_n + 2^n\}$ 是等比数列, 首项为 4, 公比为 4,

$$\therefore b_n + 2^n = 4^n,$$

$$\therefore b_n = 4^n - 2^n. (6 \text{ 分})$$

$$(3) \textcircled{1} n=2k \text{ 时}, k \in \mathbb{N}^*, c_n = c_{2k} = \frac{a_k}{b_k + 2^k} = \frac{2k-1}{4^k},$$

$$\therefore \text{数列 } \{c_{2k}\} \text{ 的前 } k \text{ 项的和 } A_k = \frac{1}{4} + \frac{3}{4^2} + \dots + \frac{2k-1}{4^k},$$

$$\therefore \frac{1}{4} A_k = \frac{1}{4^2} + \frac{3}{4^3} + \dots + \frac{2k-3}{4^k} + \frac{2k-1}{4^{k+1}},$$

$$\therefore \frac{3}{4} A_k = \frac{1}{4} + 2 \left(\frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^k} \right) - \frac{2k-1}{4^{k+1}} = \frac{1}{4} + 2 \times \frac{\frac{1}{16} \left(1 - \frac{1}{4^{k-1}} \right)}{1 - \frac{1}{4}} - \frac{2k-1}{4^{k+1}},$$

$$\text{化为: } A_k = \frac{5}{9} - \frac{6k+5}{9 \times 4^k}. (9 \text{ 分})$$

$$\begin{aligned} \textcircled{2} n=2k-1 \text{ 时}, c_n &= \frac{3 \times 2^k}{4b_k - 2^{k+1} + 2} = \frac{3 \times 2^k}{4(4^k - 2^k) - 2^{k+1} + 2} = \frac{3 \times 2^k}{(2^{k+1} - 1)(2^{k+1} - 2)} \\ &= \frac{3 \times 2^{k-1}}{(2^{k+1} - 1)(2^k - 1)} = \frac{3}{2} \left(\frac{1}{2^k - 1} - \frac{1}{2^{k+1} - 1} \right), \end{aligned}$$

$$\therefore \text{数列 } \{c_{2k-1}\} \text{ 的前 } k \text{ 项的和 } B_k = \frac{3}{2} \left[\left(\frac{1}{2-1} - \frac{1}{2^2-1} \right) + \left(\frac{1}{2^2-1} - \frac{1}{2^3-1} \right) + \cdots + \left(\frac{1}{2^k-1} \right. \right.$$

$$\left. - \frac{1}{2^{k+1}-1} \right)$$

$$= \frac{3}{2} \left(1 - \frac{1}{2^{k+1}-1} \right),$$

$$\therefore \text{数列 } \{c_n\} \text{ 的前 } 2n \text{ 项的和 } T_{2n} = \frac{5}{9} - \frac{6n+5}{9 \times 4^n} + \frac{3}{2} \left(1 - \frac{1}{2^{n+1}-1} \right). \quad (12 \text{ 分})$$